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The deep roots of solar radiance variability

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Abstract. The variability of solar radiance over a solar cycle is thought to result from a delicate balance between the radiative deficit of sunspots and the extra contribution of plage and network regions. Although the net effect is tiny, it implies structural and thermal changes in the Sun or in partial layers of it as an unavoidable consequence of the virial theorem. Using the virial theorem for continua—including the magnetic field—it can be shown how solar radiance variability might be connected to a deeply seated flux-tube dynamo and how this connection is established on a hydrodynamical time scale.

Key words. Sun – interior, Sun – magnetic fields, solar cycle

1. Introduction

It seems now well established that the immediate cause of solar radiance variability is surface magnetism (Wenzler, Solanki & Krivova 2005). Although the radiation blocking by sunspots and the extra radiation from plage and network regions are distinctly different and independent physical processes, they nearly cancel but the extra radiation slightly overcompensates the radiation blocking over a solar cycle.

An understanding of the extra radiation from plage and network regions, comes from the study of faculae. These are magnetic flux bundles that lead to a depression of the solar surface at their location, effected by the magnetic pressure. In regions where they are abundant (plage and network), they so increase the "roughness" of the solar surface, hence they increase the effective surface from where radiation can escape. A closer look to the radiation transfer in and around magnetic flux tubes reveals that not only the famed "hot wall" (Spruit 1976) contributes to the excess radiation loss, but also a wider surrounding of the flux concentration proper does (Steiner 2005). This comes about because a material parcel located at the surface sidewise of the magnetic flux element "sees" a more transparent sky in the direction of the magnetic field than in a direction away from it and under the same azimuth angle, since the flux-tube atmosphere is less dense, hence more transparent than the ambient medium. This radiative transfer effect, first obtained by Caccin & Severino (1979), makes the facula seem to be projected to the neighbouring limbward granule, hence the name "facular granules". Therefore, the cross-sectional area of excess radiative escape from faculae is much wider than the magnetic field concentration proper.

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2. Enropy flow in the convection zone

Faculae act like valves for the radiation at the solar surface so that facular regions increase the emissivity from the solar surface. They increase the cooling rate, hence produce an extra entropy deficiency of the material that flows back into the convection zone. This downflow occurs in the form of deeply penetrating, quasi adiabatic plumes. It follows that the increase in emissivity from the solar surface is, by means of an extra entropy deficient downflow, communicated to the deep convection zone on a hydrodynamical time scale. This process constitutes one aspect of the "deep roots" of solar radiance variability.

Another aspect is possibly given by an entropy rich upflow to the middle and upper layers of the convection zone that conveys information on the state of the magnetic field at the bottom. To understand this, we consider an interface dynamo in which the large-scale magnetic field is stored in the subadiabatic overshoot layer at the base of the convection zone. From this layer, strands of magnetic flux may become unstable and rise within a hydrodynamical time scale of one month to the solar surface, where they form bipolar sunspots. Computer simulations on the rise of magnetic flux tubes by different research groups and linear stability analysis conclude to a unique strength of $\approx$ 10 T this field must initially have in order to appear on the solar surface at the correct latitude and under the correct tilt angle and tilt-angle distribution with latitude (Joy's law) (D'Silva & Choudhuri 1993; Schüssler et al. 1994; Fisher et al. 2000; Moreno-Insertis et al. 1992; Ferriz-Mas & Schüssler 1995).

If the rise from the overshoot layer starts with a field less than 10 T, the flux tube gets out of pressure balance still within the convection zone because the gas pressure within the tube quickly approaches the gas pressure of the ambient medium during the rise. The tube starts to inflate and "explodes" into a cloud of weak magnetic field (Moreno-Insertis, Caligari & Schüssler 1995; Rempel & Schüssler 2001). Computer simulations show that in this highly non-linear regime an outflow along the two legs connecting to the anchored part of the tube ensues as indicated in Fig. 1. The outflow has two effects. First, with mass it removes thermal energy from the flux tube so that the tube shrinks and, keeping magnetic flux conserved, it must intensify. This process opens a venue to tap a huge potential energy reservoir for field intensification instead of the relatively limited kinetic energy of differential rotation (Rempel & Schüssler 2001). Second, the quasi adiabatic outflow injects entropy rich plasma into the middle and upper convection zone, conveying the process of magnetic intensification to these layers on a hydrodynamical time scale.

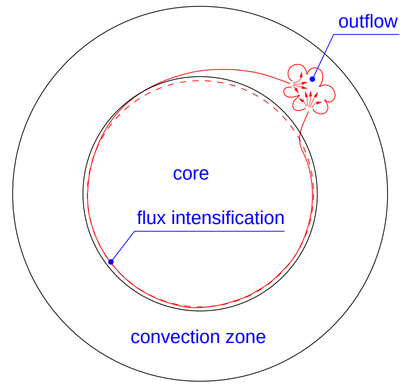

Fig. 1. Magnetic flux tubes with less than 10 T initial field strength get out of pressure balance at their apex when rising through the convection zone. An outflow of plasma ensues that leads to a magnetic intensification in the anchored part of the flux tube and to an injection of entropy rich material into the convection zone.

3. Energy estimates

From the total magnetic flux that appears in the form of sunspots over a solar cycle ($\Phi_{\rm tot} \approx 10^{16}$ Wb from Galloway & Weiss (1981)) and from the presumption that this flux must at some time exist in the from of flux tubes with a flux density of 10 T at the base of the convection zone, one obtains for the total magnetic energy generated over a solar cycle $\Delta M \approx 10^{32}$ J. On the other hand, the equivalent in thermal energy of the variation in the global solar radiation,

$$\Delta U_{\rm rad} = \int_{\rm cycle} \delta L {\rm d}t \approx \tau_{\rm cycle} \cdot 10^{-3} L_{\odot} \approx 10^{32}{\rm J}. \quad (1)$$

This "coincidence" of equal magnetic and thermal energy change was first noticed by Schüssler (1996). That the two energy variations might be in a causal relationship, is motivated by the consideration of entropy flows in Sect. 2. In Sect. 4 we try to establish a firmer connection with the help of the virial theorem.

We note that the above energy estimations are based on the assumptions that first, the solar luminosity variation over a solar cycle is of the same magnitude as the solar irradiance variation and that second, on average each sunspot pair can be associated with a magnetic flux tube that produces just one such pair.

4. Application of the virial theorem

A general form of the virial theorem including the magnetic field is given by Chandrasekhar & Fermi (1953):

$$\frac{1}{2}\frac{{\rm d}^2 J}{{\rm d}t^2} = 2K + \Omega + M + 3\int_{\mathcal{R}} p \,{\rm d}V + S, \quad (2)$$

$$J(t) = \int_{\mathcal{R}} \rho \,\|\mathbf{r}\|^2 \,{\rm d}V, \qquad M(t) = \int_{\mathcal{R}} \frac{\|\mathbf{B}\|^2}{2\mu_0}\,{\rm d}V, \quad (3)$$

$$K(t) = \frac{1}{2}\int_{\mathcal{R}} \rho \,\|\mathbf{v}\|^2 \,{\rm d}V, \qquad \Omega(t) = \frac{1}{2}\int_{\mathcal{R}} \rho \Psi \,{\rm d}V, \quad (4)$$

where J is the moment of inertia of the mass distribution in the region $\mathcal{R}$, K the kinetic energy of (macroscopic) mass motions, Ω the gravitational potential energy (with Ψ being the gravitational potential), M the magnetic energy, p the gas pressure, and S a surface integral over the boundary $\partial\mathcal{R}$ (with normal vector $\mathbf{n}$) of the region $\mathcal{R}$, viz.,

$$S(t) = -\oint_{\partial\mathcal{R}} [p_{\rm tot}(\mathbf{r}\cdot\mathbf{n}) + \frac{1}{\mu_0}(\mathbf{r}\cdot\mathbf{B})\,(\mathbf{B}\cdot\mathbf{n})]\,{\rm d}S . \quad (5)$$

Here $p_{\rm tot} = p + \|\mathbf{B}\|^2/(2\mu_0)$ is the total (gas plus magnetic) pressure. For the case of an ideal gas with constant ratio of specific heats, $\gamma = c_p/c_v$, the integral of pressure over the region $\mathcal{R}$ is related to the thermal (or internal) energy U by:

$$\int_{\mathcal{R}} p \,{\rm d}V = (\gamma - 1) U(t) \,. \quad (6)$$

The region of integration, $\mathcal{R}$, may be the entire star or part thereof. Here we will restrict attention to the convection zone, limited by two boundaries: an outer one near the solar surface, with $\|\mathbf{B}\| \approx p \approx 0$, and an inner one near the overshoot layer at the base of the convection zone but just within the radiation zone, with $\|\mathbf{B}\| = 0$. We assume that the radiative zone is not influenced by the solar cycle at all. Under this premiss, the energy production in the core remains steady and the energy flux into the domain $\mathcal{R}$ is constant. But as the radiative loss from the solar surface varies with the solar cycle, the internal energy $U(t)$ also varies, and hence, by virtue of the virial theorem [see Eq. (2)], other terms must change accordingly. One of these varying terms is supposedly the magnetic energy, M.

In Steiner (2005) and Steiner & Ferriz-Mas (2005) we argue that the left hand side in Eq. (2) is small compared to ΔM and that the kinetic term K cannot vary of the order of ΔM over a solar cycle. We would like to emphasize that this does not necessarily imply that the field amplification cannot proceed via differential rotation. But if it does, the kinetic energy of the differential rotation must be continuously replenished by a different source else it would be entirely used up over a solar cycle. This alternative source can only be either U or Ω of the convection zone, set apart the core that is presumed not to change. If a mechanism exists that feeds the differential rotation, field amplification by "flux-tube explosions" as outlined in Sect. 2 is not needed. However, the latter mechanism too, lives on U or Ω. If Ω is bound to change (which we rather believe it is not: see Sect. 6), then the surface term S counterbalances this change to first order so that $\Delta\Omega + \Delta S = 0$. This can be seen as follows.

Consider a thin shell of mass m on top of a core of mass M and radius r and assume m to be so small that its self gravitation is negligible.

Then the gas pressure at the interface between the shell and the core is

$$p_g = \frac{GMm}{r^2} \cdot \frac{1}{4\pi r^2} = \frac{GMm}{4\pi r^4}, \tag{7}$$

where $p_g = p_g(r)$ and G is the gravitational constant. Neglecting the magnetic field in the surface term

$$S = 4\pi r^3 p_g. \tag{8}$$

Together with Eq. (7) it follows:

$$\mathrm{d}S = 12\pi p_g r^2 \mathrm{d}r + 4\pi r^3 \mathrm{d}p_g = -4\pi p_g r^2 \mathrm{d}r. \tag{9}$$

But

$$\mathrm{d}\Omega = \frac{GMm}{r^2}\mathrm{d}r = +4\pi r^2 p_g \mathrm{d}r \tag{10}$$

so that

$$\mathrm{d}\Omega + \mathrm{d}S = 0. \tag{11}$$

To what extent this equation applies to the finitely thick convection zone remains to be investigated. If appropriate we are left with the reduced virial equation

$$\Delta M + 2\Delta U = 0, \tag{12}$$

where we have used Eq. (6) and have set $\gamma = 5/3$. In combination with the energy equation, Eq. (12) has far reaching consequences for the radiation from the solar surface.

5. The solar cycle in terms of the virial theorem

Starting with the half cycle over which the magnetic energy increases by

$$\Delta M = M_2 - M_1 \approx +10^{32} \text{ [J]}, \tag{13}$$

the internal energy decreases by virtue of Eq. (12), by

$$\Delta U = U_2 - U_1 = -\frac{1}{2}\Delta M. \tag{14}$$

The total energy balance between 1 and 2 reads

$$U_1 + \Omega_1 + M_1 = U_2 + \Omega_2 + M_2 + R^{\uparrow}, \tag{15}$$

where $R^{\uparrow}$ is the energy lost from the star from 1 to 2 by extra radiation from the solar surface (the energy flow entering at the bottom is presumed to be constant). From Eqs. (14) and (15) it follows that

$$R^{\uparrow} = -\frac{1}{2}\Delta M - \Delta\Omega. \tag{16}$$

If all of the magnetic energy comes from the internal energy U, then $\Delta\Omega = 0$ and $R^{\uparrow} < 0$, leading to a *reduction in radiation loss from the Sun*. If all of the magnetic energy comes from the gravitational potential we would have the opposite behaviour.

In the second half of the cycle $\Delta M \approx -10^{32}$ J because all the magnetic field gets expelled or annihilated, clearing the stage for the next cycle of opposite magnetic polarity. Then the virial equation demands

$$\Delta U = U_3 - U_2 = +\frac{1}{2}|\Delta M|. \tag{17}$$

leading in combination with the energy balance to

$$R^{\uparrow} = \frac{1}{2}|\Delta M| - \Delta\Omega. \tag{18}$$

and to an *excess of radiation from the Sun* of the order of $|\Delta M| \approx 10^{32}$ [J] if $\Delta\Omega = 0$. A reduction of radiation loss in the first half of the cycle could be caused by a reduction of the superadiabaticity, $\delta = \nabla - \nabla_{\mathrm{ad}}$, which throttles the convective energy transport because $F_{\mathrm{c}} \propto \delta$. In turn, δ is reduced by the excess of entropy injected into the convection zone by the magnetoconvective process of flux-tube rise and flux-tube explosion. Alternatively, if convective motion is doomed to replenish differential rotation for the generation of magnetic energy, a corresponding reduction in convective energy transport and hence in luminosity can be expected too. On the other hand, excessive radiation loss in the second half of the cycle is caused by radiative channeling in magnetic elements of network and plage regions.

6. Source of the magnetic energy

Whether the magnetic energy ultimately stems from the gravitational or thermal potential energy of the convection zone we can only speculate. Note, however, what matters energetically is the last stage of field intensification,

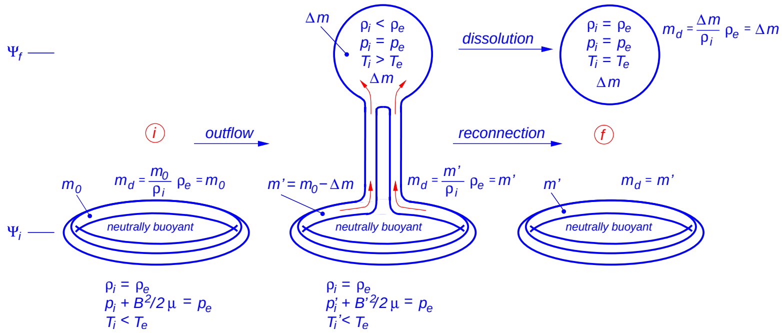

Fig. 2. Schematic of flux-tube intensification by mass removal. The gravitational energy in the initial and final state are the same.

say from 1 to 10 T, viz., the generation of the large scale magnetic flux tubes.

Assuming first, that this field is immediately generated by differential rotation, the latter would have to be efficiently replenished, probably via shear forces by convective motion. This would rather decrease the convective energy transport, hence decrease the solar luminosity and leading rather to an expansion of the convection zone, hence an increase in the gravitational potential energy. In this case $\Delta\Omega \geq 0$ in Eq. (16) hence $R^{\uparrow} < 0$ in accordance with the assumption. The magnetic energy would have ultimately to come from the internal energy U.

The schematics of Fig. 6 on the other hand illustrates the case of flux intensification by "flux-tube explosion". Starting from an initial flux tube located at gravitational potential Ψ_i, mass Δm is transferred from the flux tube to a bubble at potential Ψ_f. Thus, mass and with it thermal energy is removed from the initial flux tube and partially replaced by magnetic energy through intensification. In an intermediate state the mass in the bubble is at higher temperature and lower density than the ambient medium (because of the superadiabatic stratification of the background atmosphere) but with dissolution the difference disappears on a hydrodynamical time scale so that the mass displaced by Δm equals Δm. Striking the balance between initial and final state one can see that the mass moved from potential Ψ_i to Ψ_f is exactly balanced by the displace mass moved in the opposite direction, so that the total gravitational potential of the initial and final configurations are identical. In this case too, the source of the magnetic energy is the internal energy of the ambient medium.

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