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Connecting solar radiance variability to the solar dynamo with the virial theorem

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Abstract. The variability of solar radiance over a solar cycle is thought to result from a delicate balance between the radiative deficit of sunspots and the extra contribution of plage and network regions. Although the net effect is tiny, it must imply structural and thermal changes in the Sun or in partial layers of it as an unavoidable consequence of the virial theorem. Using the virial theorem for continua—including the magnetic field—it can be shown how solar radiance variability might be connected to a deeply seated flux-tube dynamo and how this connection is established on a hydrodynamical time scale.

Key words: Sun: interior – Sun: magnetic fields – Sun: activity cycle

1. What can the virial theorem tell us?

A general form of the virial theorem including the magnetic field is given by Chandrasekhar & Fermi (1953):

$$\frac{1}{2}\frac{\mathrm{d}^2 J}{\mathrm{d}t^2} = 2K + \Omega + M + 3\int_{\mathcal{R}} p \mathrm{d}V + S\,, \tag{1}$$

$$J(t) = \int_{\mathcal{R}} \rho\,\|\mathbf{r}\|^2\,\mathrm{d}V\,, \qquad M(t) = \int_{\mathcal{R}} \frac{\|\mathbf{B}\|^2}{2\mu_0}\,\mathrm{d}V\,, \tag{2}$$

$$K(t) = \frac{1}{2}\int_{\mathcal{R}} \rho\,\|\mathbf{v}\|^2\,\mathrm{d}V\,, \qquad \Omega(t) = \frac{1}{2}\int_{\mathcal{R}} \rho\Psi\,\mathrm{d}V\,, \tag{3}$$

where J is the moment of inertia of the mass distribution in the region $\mathcal{R}$, K the kinetic energy of (macroscopic) mass motions, Ω the gravitational potential energy (with Ψ being the gravitational potential), M the magnetic energy, p the gas pressure, and S a surface integral over the boundary $\partial\mathcal{R}$ (with normal vector $\mathbf{n}$) of the region $\mathcal{R}$, viz.

$$S(t) = -\oint_{\partial\mathcal{R}} p_{\mathrm{tot}}(\mathbf{r}\cdot\mathbf{n})\,\mathrm{d}S + \frac{1}{\mu_0}\oint_{\partial\mathcal{R}} (\mathbf{r}\cdot\mathbf{B})\,(\mathbf{B}\cdot\mathbf{n})\,\mathrm{d}S\,. \tag{4}$$

Here $p_{\mathrm{tot}} = p + \|\mathbf{B}\|^2/(2\mu_0)$ is the total (gas plus magnetic) pressure. For the case of an ideal gas with constant ratio of specific heats, $\gamma = c_p/c_v$, the integral of pressure over the region $\mathcal{R}$ can be related to the thermal (or internal) energy U:

$$\int_{\mathcal{R}} p\,\mathrm{d}V = (\gamma - 1)U(t)\,. \tag{5}$$

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The region of integration, $\mathcal{R}$, may be the entire star or part thereof. Here we will restrict attention to the convection zone, limited by two boundaries: an outer one near the solar surface, with $\|\mathbf{B}\| \approx p \approx 0$, and an inner one near the overshoot layer at the base of the convection zone but just within the radiation zone, with $\|\mathbf{B}\| = 0$ and $p = \mathrm{const}$, so that $S(t) = \mathrm{const}$. Here we assume that the radiative zone is not influenced by the solar cycle at all. The condition $p = \mathrm{const}$, that holds strictly only for a constant gravitational field within the convection zone, will be revisited at the end of Sect. 2. Under this assumption, the energy production in the core remains steady and the energy flux into the domain $\mathcal{R}$ is constant. But as the radiative loss from the solar surface varies with the solar cycle, the internal energy $U(t)$ also varies, and hence, by virtue of the virial theorem [see Eq. (1)], other terms must change accordingly. One of these varying terms is supposedly the magnetic energy, M. Together with the total energy equation, the virial theorem may thus help us to connect the cyclic variation of the magnetic energy, i.e., dynamo action, to the cyclic variation of the solar radiance.

2. Estimates for the various virial terms

The region $\mathcal{R}$ encompasses the convection zone, for which we estimate in the following the variation of the various virial terms over the solar cycle.

The variation of the *magnetic energy* is estimated by noting that one solar cycle generates a magnetic flux of $\Phi \approx 10^{16}$ Wb (Galloway & Weiss, 1981). We assume that this flux is generated by an interface dynamo located near the base of the convection zone. If the flux resides in the overshoot layer in the form of a flux sheet with a strength of 10 T, it has a thickness of $d \approx 10^6$ m with a total magnetic energy of

$$M \approx 5 \times 10^{32}\,[\mathrm{J}]\,, \tag{6}$$

where the latter number is independent of the filling factor (Rempel, 2001). This number must be considered an upper limit because it does not take into account the fact that a single flux tube after eruption could possibly reconnect and erupt again as a different active region so that its magnetic flux content would be counted twice at the solar surface.

Thinking in terms of a dynamo wave, one might argue that M must remain constant (Tobias, personal communication). However, the last stage of field intensification that energetically matters, say from 1 T to 10 T, is thought to take place by a non-linear process, superposed on the linear dynamo wave, as explained in more detail further below.

The second time derivative of the *moment of inertia* is estimated as follows:

$$\mathrm{d}^2 J/\mathrm{d}t^2 \approx J/\tau_{\mathrm{cyc}}^2 \approx 10^{-3} \times M\,. \tag{7}$$

It is much smaller than the magnetic energy and can be neglected, meaning that the system is in a relaxed state.

The total *kinetic energy* consists of two contributions: convective motions and rotation. The kinetic energy of convective motions in the total convection zone is (taking velocities from a mixing length model):

$$K_{\mathrm{con}} = (1/2)\int_{\mathcal{R}} \rho\,\mathbf{v}_{\mathrm{con}}^2\,\mathrm{d}V = 7.8\times 10^{31}\,[\mathrm{J}] = 0.16\times M\,. \tag{8}$$

Also, because convective heat transport to the surface cannot vary more than the solar luminosity does, and because $F_{\mathrm{conv}} \propto v_{\mathrm{conv}}^2$, $\delta F_{\mathrm{con}}/F_{\mathrm{con}} = \delta K_{\mathrm{con}}/K_{\mathrm{con}} < \mathcal{O}(10^{-3})$.

The kinetic energy of rotation in the convection zone is:

$$K_{\mathrm{rot}} = (1/2)\int_{\mathcal{R}} \rho[\Omega(\theta,\mathbf{r}) \times \mathbf{r}]^2\,\mathrm{d}V = 110 \times M\,. \tag{9}$$

However, the available kinetic energy for variation over the solar cycle is only that from differential rotation, i.e., the difference in kinetic energy between differential rotation and rigid rotation with equal rotational momentum. It is

$$K_{\mathrm{diff}} = 1 \times 10^{33}\,[\mathrm{J}] = 2 \times M\,. \tag{10}$$

Helioseismic observations reveal a “torsional oscillation” in phase with the solar cycle with an amplitude of 0.04 degrees/day, which is two orders of magnitude smaller than differential rotation. Thus, its energy is two orders of magnitude smaller than K_{diff}. Any variation larger than this would have long since been discovered. Therefore, we set as a hypothesis

$$K(t) = \mathrm{const.} \tag{11}$$

While kinetic energy is assumed not to be a major source for field intensification, differential rotation is certainly an important agent for generating the ($\lesssim 1\,\mathrm{T}$) toroidal field in the first place. Torsional oscillation need not necessarily be a manifestation of Lorentz forces acting on the plasma. Spruit (2003) concludes that it is due to a geostrophic flow driven by temperature variations at the surface, which in turn are a consequence of solar radiance variability.

The variation of the *thermal energy* can be estimated by an integration of the solar luminosity variation over a solar cycle, $\delta L \approx 0.001 \times L_{\odot}$:

$$\Delta U_{\mathrm{rad}} = \int_{\tau_{\mathrm{cycl}}} \delta L\,\mathrm{d}t \approx 10^{32}\,[\mathrm{J}]\,. \tag{12}$$

The coincidence of equal magnetic and thermal energy change over a solar cycle seems to have been first noticed by Schüssler (1996). The idea here is to obtain a connection between the two variations with the help of the virial theorem.

Considering a convection zone of small mass on top of a radiative core of large mass with radius r, it can be readily shown that a small variation in r leads to a change in the *surface term* of $\delta S = -4\pi r^2 p_g(r)\delta r$. In order that this term does not grow important, $\delta S \ll 10^{32}$ [J], which translates to $\delta r \ll 5$ [m]. However, such a variation would cause a corresponding variation in Ω of exactly the same size but of opposite sign so that the two terms cancel and they can be expected to cancel to leading orders for larger variations of r. To what extent such a cancellation also holds for other variations than a simple translation, e.g., for a change in the superadiabaticity through the convection zone, remains to be investigated (but also see remark after Eq. (15) and (16)).

The gravitational binding energy will be discussed in Sect. 4. Here we note that the absolute value of Ω and U for the total convection zone is of the order of 10^{40} J. Considering variations over a solar cycle, three significant terms of the full virial equation remain:

$$\Omega + M + 2U = 0\,, \tag{13}$$

where we have set $\gamma = 5/3$. A possible scenario that relates the three terms is provided by a flux-tube dynamo operating in the overshoot layer at the base of the convection zone.

3. Flux intensification and entropy transport by large-scale flux tubes

In the flux-tube dynamo picture (see, e.g., Schüssler & Ferriz-Mas, 2003), the large-scale magnetic field is generated in the overshoot layer beneath the convection zone, where the toroidal flux can be stored in mechanical equilibrium owing to the subadiabatic stratification. From there, strands of magnetic flux may undergo an undular instability so that magnetic loops start rising through the convection zone (Fig. 1); if they reach the photosphere, they ultimately appear as bipolar sunspot pairs at the solar surface. The time scale for the rise through the convection zone is of the order of 1 month.

In order that sunspots appear at the correct latitude, with the correct tilt angle between preceding and following spots and, further, the correct dependency of the tilt angle on latitude (Joy’s law), the flux tubes must have a field strength of about 10 T at the point of departure of their journey at the base of the convection zone. This value is the outcome of both numerical simulations of the rise of large-scale flux

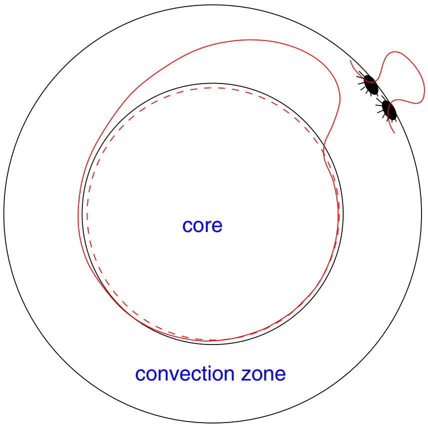

Fig. 1. (online colour at www.an-journal.org) Magnetic flux tube at the base of the convection zone (dashed circle), rising through the convection zone to the surface (solid red curve), where it forms a sunspot pair.

tubes through the convection zone (e.g., D'Silva & Choudhuri, 1993; Schüssler et al., 1994; Fisher et al., 2000) and of a linear stability analysis of toroidal flux tubes (Moreno-Insertis et al., 1992; Ferriz-Mas & Schüssler, 1995). But what physical mechanism is able to intensify the field strength up to 10 T? Moreno-Insertis et al. (1995) and Rempel & Schüssler (2001) have shown that flux-tube rise can also be a mechanism for field intensification.

3.1. Intensification and entropy transport by flux-tube explosions

Consider an erupting flux tube that has not yet achieved the required field strength of 10 T. If this flux tube rises adiabatically through the convection zone, its gas pressure, $p_{\rm i}$, decreases less rapidly than that of the superadiabatically stratified ambient medium, $p_{\rm e}$. Hence, there is a critical height, $z_{\rm exp}$, at which the magnetic pressure must vanish and the flux tube "explodes" into a cloud of weak field. For an initial field strength < 10 T, this height may be located well within the convection zone.

As a consequence of the flux-tube explosion, mass flows upwards into the convection zone from the part of the flux tube that remains anchored below the convection zone; this leads to a partial evacuation of the flux tube and, in consequence, to an increase in flux density (and hence in magnetic energy). This effect, which is contrary to what happens in the initial phase of the Parker instability, occurs in the non-linear phase of flux-tube rise (Moreno-Insertis et al., 1995; Rempel & Schüssler, 2001). Another consequence of this process is that entropy-rich plasma from the base of the convection zone flows into upper layers. This process takes place on a hydrodynamical time scale of approximately one month. Rempel (2001) estimates that the energy carried by this entropy flux over one solar cycle is $\Delta U \approx 1 \times M$. This result must be expected since the energy flux must correspond to the increase in magnetic energy, which in turn we know to be of the order of $\int_{\rm cycle} \delta L {\rm d}t$.

The process of flux-tube rise and flux-tube explosion can be interpreted as a magnetoconvective mixing-length transport, similar to, but more efficient than, the regular hydrodynamic one because the "magnetoconvective mixing length" can be a large fraction of the convection-zone depth. By this process, the convection zone finds a more efficient means of transporting energy across it, limited only by the amount of available magnetic flux.

Instead of the explosion mechanism, we could think of an "evacuation" of flux tubes by a leakage of plasma (Brummell, 2004) that would take place *locally* and that could possibly lead to an 'adiabatization' of the overshoot layer with a subsequent change in the superadiabaticity throughout the convection zone.

4. The solar cycle in terms of the virial theorem

Starting with a weak toroidal field of $B \approx 1$ T, this field gets amplified in the course of the solar cycle to 10 T by the process of flux-tube explosions, increasing the field energy by the amount

$$\Delta M = M_2 - M_1 \approx 10^{32} \text{ [J]} , \tag{14}$$

so that, according to the virial equation, Eq. (13),

$$\Delta U = U_2 - U_1 = -\frac{1}{2}(\Delta\Omega + \Delta M) . \tag{15}$$

Except for the rising loops, the flux tubes stay in mechanical equilibrium; i.e., they are neutrally buoyant during the intensification process, which suggests $\Delta\Omega \approx 0$. If $\Delta\Omega = -\Delta M$, then $\Delta U = 0$, which means that no luminosity variation would be induced. But in this case we would expect a non-negligible positive surface term, which would lead to $\Delta U = -(1/2)|\Delta S|$. Therefore we think that at least part of the magnetic intensification lives on a reduction of the internal energy potential. Such a reduction might ensue as a consequence of the reduction in superadiabaticity by the increased efficiency of convective energy transport as explained in Sect. 3.1. Still lacking a rigorous proof, we propose $\Delta\Omega + \Delta M > 0$. With this proposition, Eq. (15) leaves an internal energy deficiency of -10^{32} J, leading to a *reduction in radiation loss from the sun*, in fulfillment of the total energy equation, viz.

$$E_{\rm tot1} = U_1 + \Omega_1 + M_1 = E_{\rm tot2} = U_2 + \Omega_2 + M_2 + R^{\uparrow} , \tag{16}$$

where $R^{\uparrow}$ is the energy lost from the star, presumably by extra radiation from the solar surface. A reduction of radiation loss, i.e., a negative $R^{\uparrow}$, could be caused by a reduction of the superadiabaticity, $\delta = \nabla - \nabla_{\rm ad}$, which throttles the convective energy transport because $F_{\rm c} \propto \delta$. In turn, δ is reduced by the excess of entropy injected into the convection zone by the magnetoconvective process of flux-tube rise and flux-tube explosion. Such an 'adiabatization' would lead to an expansion of the atmosphere, hence to a positive contribution to $\Delta\Omega$ in Eq. (15) (if not compensated for by the surface term) and hence, in agreement with our proposition that $\Delta U < 0$.

In the second half of the cycle $\Delta M \approx -10^{32}$ J because all the magnetic field gets expelled or annihilated, clearing the stage for the next cycle of opposite magnetic polarity. This is the time of magnetic activity at the solar surface. Again, assuming $\Delta\Omega \approx 0$ (or $\Delta\Omega < 0$ in order to obtain $\Delta U > 0$), Eq. (13) leads to

$$\Delta U = -\frac{1}{2}(\Delta\Omega + \Delta M) \approx +10^{32} \text{ [J]} > 0 , \tag{17}$$

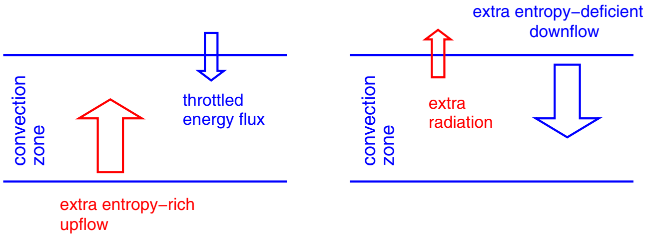

Fig. 2. (online colour at www.an-journal.org) The thermodynamic cycle associated with the magnetic solar cycle.

leaving an internal energy excess of 10^{32} J, which must be lost by an *excess of radiation from the Sun* in fulfillment of Eq. (16).

Excessive radiation loss is caused by radiative channeling in magnetic elements of network and plage regions. But excessive cooling entails entropy-deficient material beneath these regions being transported back to the depths of the convection zone by the downflows. A pictorial interpretation of the thermodynamic ideas sketched above is illustrated in Fig. 2. The magnetoconvective transport of entropy-rich material into the convection zone, with the associated reduction of convective energy transport (1st stage), and the formation of plage and network faculae, with the associated transport of entropy-deficient material back into the convection zone (2nd stage)—all these processes work on a hydrodynamic timescale, so that structural and radiative changes caused by the dynamo process take place on the hydrodynamical, and not the Kelvin-Helmholtz time scale.

5. Discussion

In reality one cannot sharply divide the solar cycle into the two stages depicted in Fig. 2. Flux intensification and flux expulsion/annihilation can proceed at the same time, depending on the phase relation between flux generation and flux removal, where the latter must not be identical to the rate of flux emergence at the solar surface.

Furthermore, U and M need not necessarily be out of phase by exactly $180°$ but can be so by much less than $90°$, provided that Ω changes correspondingly. In this case the luminosity modulation would be smaller than the amplitude of M, which is suggested to be so by our estimation given in Sect. 2, where $M/\int_{\rm cycle}\delta L {\rm d}t \approx 5$.

It is important to note that the radiance variation is tightly associated with the surface magnetism because it provides —through network and plage regions—the necessary means for enhanced radiation from the surface. Accordingly, there is no phase shift observed between radiance variation and indices of surface magnetism such as the sunspot number. Any phase lag between radiance and magnetic energy different from $180°$ must be absorbed by a corresponding change in the gravitational binding energy, irrespective of whether flux intensification does or does not require gravitational potential energy. Thus, surface magnetism and dynamo action are the driver of the thermodynamic cycle, while the gravitational binding energy provides the reservoir potential for any discordance with the combined virial and energy equation.

The phase relations among M, ΔU and δL have been obtained from numerical simulations of an axisymmetric compressible mean-field dynamo by Brandenburg et al. (1992). There, δL lags M by $43°$, which is not in contradiction with the theory presented here. However, they also show δL and the variation of the internal energy to be out of phase by approx. $180°$, which hints that other terms of the virial equation such as the surface term come into play in their simulation.

In contrast to Spiegel & Weiss (1980), who thought the cause of luminosity variation to be in the magnetic field at the base of the convection zone directly, we believe that it is immediately caused by surface magnetism. But its consequences are communicated on a hydrodynamical time scale to the base of the convection zone by the downward transport of entropy deficient material, which is balanced by the transport of entropy rich material back into the convection zone by the flux-tube explosion mechanism. As does the magnetic field in Spiegel & Weiss (1980), both of these processes change the superadiabaticity in the convection zone.

If solar radiance variability would influence only the top layer of the convection zone, the virial theorem could be used to compute the change in the density scale height to account for the variation in the gravitational binding energy over the solar cycle. If this depth were 10 Mm, and H_ρ were independent of height, ΔH_ρ would be 10 km. With a depth of 1 Mm it would vary by 370 km.

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