Solar Radiance Variability as a Direct Consequence of the Flux–tube Dynamo

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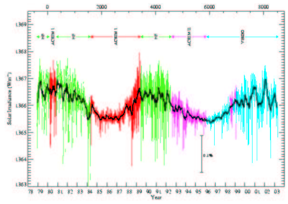
1. What does the virial theorem tell us?

The total solar irradiance ("solar constant") varies in phase with the solar cycle by ≈ 0.09%, peak-to-peak. Assuming that this variability approximately also applies to the solar luminosity, then, as an *unavoidable consequence of the virial theorem*, it implies structural changes of the Sun or partial layers of it on that same time scale. What kind of structural changes may it be?



Composite of the daily values of the total solar irradiance, plotted in different colors for the different experiments. From C. Fröhlich, www.pmodwrc.ch

A *general form of the virial theorem* for continua including the magnetic field, is given by Chandrasekhar & Fermi (1953):

$$\frac{1}{2}\frac{d^2J}{dt^2} = 2K + \Omega + M + 3\int_{\mathcal{R}} P dV + S\,,$$

where J is the moment of inertia, K the kinetic energy of mass motion, M the magnetic energy, Ω the gravitational binding energy of the star, and S a surface integral over the boundary $\partial\mathcal{R}$ of the region $\mathcal{R}$ occupied by the star. This surface integral vanishes if the magnetic field at $\partial\mathcal{R}$ is either perpendicular to $\partial\mathcal{R}$ or very weak, which, at the solar surface, is both approximately fulfilled.

A variation of the total internal energy (as a consequence of the luminosity variation) goes along with corresponding changes in one or several of the other virials, one of which, e.g., being the total magnetic energy that varies with the solar cycle.

2. Significance of the various virials

The magnetic virial: One solar cycle generates a magnetic flux of $\Phi \approx 10^{16}$ Wb. If this flux resides in the overshoot layer in the form of flux tubes with a strength of 10 T, it has a total magnetic energy of $\boxed{M \approx 10^{32}\ \mathrm{J}}$.

The kinetic virial: This magnetic energy is mainly generated in the last stage of the intensification process, starting from a magnetic layer of initial flux density 1 T within a latitude belt of ±30°. It would have a thickness of 10^7 m for to harbour 10^{16} Wb magnetic flux, within a volume of 3×10^{25} m^3. Within this volume, the available kinetic energy of convective motion is $\boxed{K \approx 0.1\times M}$. Also, from mixing-length theory, the convective energy transport is $F_C \propto v_c^3$, from where it follows that v_c^2 may not vary more than 10^{-3} over the solar cycle. The kinetic energy of convective motion in the whole convection zone is $1.8 \times M$, thus, $\boxed{\Delta K < 2\times10^{-3}\times M}$. The kinetic energy that can be drawn from differential rotation keeping the angular momentum constant is again $\boxed{K_{\mathrm{diff}} < 0.1\times M}$, while we assume that there exists no external torque so that $\boxed{\Delta K_{\mathrm{rot}} \approx \mathrm{const.}}$. Note, however, that for the entire convection zone $K_{\mathrm{rot}} \approx 500\times M$.

The inertial virial: In that same volume, the potentially available momentum is $\boxed{d^2J/dt^2 \approx J/\tau_{\mathrm{cyc}}^2 \approx 10^{-3}\times M}$.

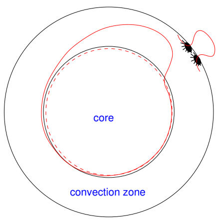
While convective motion is an unlike source for the last stage of flux intensification because this intensification proceeds on a global scale, differential rotation would have to be very efficiently replenished for feeding the required energy to the magnetic field. Lacking such a mechanism, only the *three* middle *virial terms* on the r.h.s of the full virial equation are *significant*:

$$\boxed{\Omega + M + 2U = 0\,. \qquad (*)}$$

A *possible scenario* that relates the three terms Ω, M, and U over a solar cycle is provided by the *flux-tube dynamo*.

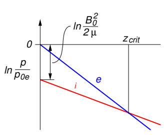
3. Mass and entropy transport by large-scale magnetic flux tubes

In the flux-tube dynamo, magnetic field is generated near the bottom of the convection zone, from where it rises to the solar surface in a 11 year period.



Magnetic flux tube at the bottom of the convection zone (dashed circle), rising through the convection zone to the surface (solid red curve), where it forms a sunspot pair.

As flux tubes rise through the convection zone, the gas pressure decreases more rapidly in the superadiabatic external atmosphere, p_e, than it does in the adiabatic flux-tube atmosphere, p_i. At the critical height, $z_{\mathrm{crit}} = z_{\mathrm{exp}}$, the magnetic pressure must vanish and the flux tube "explodes" to a cloud of weak field.



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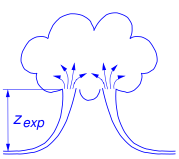

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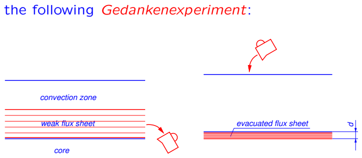
By the process of *flux-tube eruption and explosion*, entropy-rich plasma gets transported from the bottom, and dispersed to upper layers of the convection zone on a time-scale of one month.

One can easily estimate, that this entropy transport leads to a *variation of the internal heat content* of the convection zone over one solar cycle of $\boxed{\Delta U \approx 1\times M}$.

As a consequence of the flux-tube explosion, mass gets "sucked" out of the flux tube buried in the overshoot layer at the bottom of the convection zone, which leads to an increase in flux density, thus to an increase in magnetic energy (→ Sect. 2).

Another consequence is a slight lifting of the convetion zone, leading to a *change in the gravitational potential energy,* Ω. To see this, we make the following *Gedankenexperiment*:



The mass in a layer of magnetic field, sufficiently weak to have no effect on the structure of the convection zone, is bottled (left) and returned on the top to the convection zone (right). It results an evacuated flux sheet of finite thickness, d, below a convection zone that is identical to the initial one but lifted by a distance of d.

In reality, the flux sheets get not evacuated but amplified to a maximal field strength of $B_{\max} = 10^5$ G, corresponding to a plasma beta of $\beta \approx 10^5$. The corresponding change in Ω is:

$$\boxed{\Delta\Omega\ = p_{\mathrm{mag}}\cdot A\cdot d \approx 10^{32}\ \mathrm{J} \approx M\,,}$$

where p_{mag} is the magnetic pressure within the flux sheet, and A its area, which extends over a latitude belt of ±30°.

4. The solar cycle in terms of the virial theorem

Starting with a weak toroidal field of $B \approx 1$ T, we designate with Ω_1, M_1, and U_1 the initial virials of the potential gravitational, the magnetic, and the internal energy, respectively.

This field, located in a sheet near the bottom of the convection zone, gets amplified to 10 T by the process of "flux-tube" explosions, which (→ see Sect. 3) is accompanied by an increase in the gravitational potential energy,

$$\begin{aligned}\Delta\Omega &= \Omega_2 - \Omega_1 \approx 10^{32}\ \mathrm{J} > 0\,,\\ \Delta M &= M_2 - M_1 \approx M \approx 10^{32}\ \mathrm{J} > 0\,,\end{aligned}$$

so that, according to Eq. (*),

$$\Delta U = U_2 - U_1 = \frac{1}{2}(\Delta\Omega + M) \approx -10^{32}\ \mathrm{J} < 0\,,$$

leaving an internal energy deficiency of another 10^{32} J, which must be gained by a *reduction in radiation loss from the Sun* in fulfillment of the stability criterion $E_{\mathrm{tot}} \leq 0$.

A reduction of radiation loss is caused by a reduction of the superadiabaticity, $\delta = \nabla - \nabla_{\mathrm{ad}}$, which throttles the convective energy transport, $F_C \propto \delta$ (less negative entropy transport in the downward direction by plumes). δ is reduced by the excess of entropy that is transported into the convection zone by the magnetoconvective process of flux-tube rise and flux-tube explosion.

In a third stage, all the magnetic field gets ejected from the bottom of the convection zone, clearing stage for the next solar cycle with opposite magnetic sign. This is the time of magnetic activity at the solar surface. The convection zone settles back to $\Omega_3 = \Omega_1$. Thus,

$$\begin{aligned}\Delta M &= M_3 - M_2 = -M \approx -10^{32}\ \mathrm{J} < 0\,,\\ \Delta\Omega &= \Omega_3 - \Omega_2 \approx -10^{32}\ \mathrm{J} < 0\,,\end{aligned}$$

so that, according to Eq. (*),

$$\Delta U = U_3 - U_2 \approx +10^{32}\ \mathrm{J} > 0\,,$$

leaving an internal energy excess of 10^{32} J, which must be lost by an *excess of radiation from the Sun*.

Excessive radiation loss is caused by radiative channeling in magnetic elements of network and plage regions. The excessively cooled surface material transports entropy deficient material in the form of downflow plumes back to the depth of the convection zone.

Note, the magnetoconvective transport of entropy-rich material into the convection zone (1st stage), the reduction of convective energy transport (2nd stage), and the transport of entropy-deficient material back into the convection zone (3rd stage), all processes work on a hydrodynamic time-scale, so that the response for structural changes caused by the dynamo process takes place on the hydrodynamical, not the Kelvin-Helmholtz time-scale.

5. Conclusions

Solar radiance variability naturally fits into the global picture of the solar dynamo. The virial theorem provides the tool for understanding the corresponding cyclic sequence of cause and action. Accordingly, solar radiance variability is tightly connected to the magnetic field at the solar-surface but it originates deep in the convection zone. The connection to the dynamo is provided by processes that work on a hydrodynamic time-scale. These are the transport (bottom up) of extra entropy-rich material by buoyant global magnetic flux-tubes , and the transport (top down) of extra entropy-deficient material from plage and network regions by downflow plumes.

The present theory provides an explanation for the the hitherto "coincidence" of the magnetic energy generated over one solar cycle approximately equating the variation of solar luminosity integrated over the same period.

Observational consequences might bear the predicted variation of the superadiabaticity, δ, with the solar cycle, possibly better observable on stars with a magnetic cycle of large amplitude.